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A STATISTICAL ANALYSIS OF TRANSFORMATION METHODS FOR WIND POWER CURVE MODELLING

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ABSTRACT

The dissemination of wind speed is the foundation for the calculation of wind energy potential necessary for the design of wind farms. Thus, a perfect determination of the probability distribution of wind speed is an imperative parameter to measure before estimating the wind energy potential over a particular region. In this paper 10 different distributions have been compared to fit the wind speed data. The goodness of fit of the wind speed was analysed by Kolmogorov-Smirnov test, Anderson Darling test and Chi-square test. To avoid the practical difficulties in obtaining the wind power data of different stations, two transformation methods have been proposed to calculate the wind power from the observed wind speed data of the particular station. This paper describes and compares the wind power modelling using two different transformation methods namely four parameter logistic and five parameter logistic power expressions. The constants of these expressions are evaluated using the Differential Evolution (DE) and Particle Swarm Optimization (PSO) algorithms. The powers obtained by the two transformations are compared with empirical power, data and their performance analysed using the error metrics MAD & RMSE. The results indicate that the five parameter logistic (DE) transformation method is the better method to evaluate the power from the observed wind speed data.

KEYWORDS: 4P Logistic, 5P Logistic, Weibull, Burr (4P) & Wind Power Density Curve

Original Article

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INTRODUCTION

Renewable energy from the wind resource gives a favourable solution to the world which is endangered by an energy calamity and further ecological calamities. In the direction of making wind energy a consistent basis of energy, proficient and perfect models for perceiving and predicting of wind power is very essential. Wind power produced from a wind farm largely be influenced by the wind speed [1]. The association concerning the speed of the wind and the power produced by it is illustrated by the wind power curve.

The theoretical power equation is given as $P = \frac{1}{2}\rho\pi r^2 C_p(\lambda, \beta)x^3$ (1)

Where P is the power captured by the rotor of a wind turbine, ρ is the air density, r is the radius of the swept area, C_p is the power coefficient, β is the blade-pitch angle, λ is the tip-speed ratio and x is the wind speed [2].

Wind power density curves predominantly designate the performance of wind farms. Therefore evolving exact power curve models is an imperative part of research. A perfectly modeled wind power curve can also function as a device for forecasting. Modelling of power curves are useful to the expansion of power farms with

wind power [3]. Wind power density curve models are categorized into parametric and nonparametric models. Compared to other models, the parametric least squares model and the nonparametric k-nearest neighbour model is more accurate [2]. A cluster center based fuzzy logic model for wind power has been introduced in [5]. It was also recommended that more precise modelling may attained possibly by increasing the number of clusters. Based on statistical tools, the comparison of five different power curve models have been done and the better conclusions are obtained by using the fuzzy logic tool in [6]. A critical examination of the numerous power curve modeling methods like weibull distribution, method of least squares, and cubic spline interpolation have been compared in [8].

In this paper, transformation methods are applied to model the wind power density curve from the observed wind speed data. 4P logistic and 5P logistic expressions have been suggested to derive the wind power. The constants of these expressions are obtained from DE and PSO algorithms. The wind power density curve is modelled by three sets of data. Comparison of the observed wind power data and the derived wind power from the transformation methods also have been done using the error metrics MAD and RMSE.

TRANSFORMATION METHODS IN MODELLING WIND POWER

The wind power can be estimated from the wind speed data using the power equation (1). Practically it is difficult to obtain the power data from the wind farm. To avoid this difficulty, an alternative method of transformation of random variables is introduced. Using these transformation methods wind power can be evaluated from the observed wind speed data itself. Two different transformation methods are discussed and powers are derived from the observed wind speed data itself. It is a commonly used technique in statistical analysis to derive the pdf for the function of random variable, h(X).

Let P = h(X) (2)

where X is a random variable for wind speed data and $f_X(x)$ is pdf of X. Then the power pdf can be derived by

$$f_P(p) = f_X\left(h^{-1}(P)\right)\left|\frac{d\left[h^{-1}(P)\right]}{dP}\right| \text{ , } p \in P \tag{3}$$

$$= 0 \qquad\qquad , \text{otherwise}$$

- 4P Logistic Transformation

The four parameter logistic transformation to derive power is given by

$$P = a\left(\frac{1+me^{-X/\tau}}{1+ne^{-X/\tau}}\right) \tag{4}$$

Here a, m, n, τ are the constants. Hence from (2) we get $h^{-1}(P) = X$

Thus $h^{-1}(P) = T\log\left(\frac{Pn-am}{a-P}\right)$

$$= T\left[\log(Pn-am)-\log(a-P)\right]$$

$$\frac{d}{dp}\left(h^{-1}(P)\right)=T\left[\frac{n}{Pn-am}+\frac{1}{a-P}\right]$$

Thus by substituting this in equation (3), we can derive the wind power pdf $f_p(p)$.

- 5P Logistic Transformation

The five parameter logistic transformation to derive power is given by

$$P=d+\frac{(a-d)}{\left(1+\left(\frac{x}{c}\right)^{b}\right)^{g}} \tag{5}$$

Here a,b,c,d,g are the constants. Hence from (2) we get $h^{-1}(P) = X$

Thus $h^{-1}(P)=c\left[\left(\frac{a-d}{P-d}\right)^{1/g}-1\right]^{1/b}$

$$\frac{d}{dP}\left(h^{-1}(P)\right)=\frac{c}{b}\left[\left(\frac{a-d}{P-d}\right)^{1/g}-1\right]^{\frac{1}{b}-1}\cdot\frac{1}{g}\left(\frac{a-d}{P-d}\right)^{\frac{1}{g}-1}\cdot\left(\frac{d-a}{(P-d)^{2}}\right)$$

The constants of four and five parameter logistic transformations can be solved by several advanced algorithms GA, EP, PSO and DE. As the DE and PSO algorithms are ranked better [], we have used these algorithms to determine the above constants.

WIND SPEED MODELLING

However we have several PDF's, very few are appropriate for the physical data. In this study, selected ten distributions are compared which are more applicable to the perseverance. They are: Burr, Burr(4P), Dagum, Inverse Gaussian, Log-Logistic, Logistic, Lognormal, Nakagami, Rayleigh and Weibull distributions. The parameters of the above distributions are measured using the Maximum Likelihood Estimation (MLE) and given in Table 1. Figure 1 shows the fitting of observed wind speed data of various distributions.

Table 1: Parameter Estimation Using MLE

Distribution	Station 1	Station 2	Station 3
Burr	k=9.6078 α=2.5569 β=16.866	k=143.51 α =2.0743 β =98.154	k=734.64 α =2.5737 β =132.43
Burr (4P)	k=9.2761 α =2.5759 β =16.587 γ=-0.02819	k=235.34 α =1.8553 β =155.14 γ =0.67975	k=150.92 α =2.6902 β =68.412 γ =-0.39263
Dagum	k=0.28672 α =6.9894 β =8.8139	k=0.46095 α =4.2951 β=9.9565	k=0.12375 α =13.616 β =14.42
Inv. Gaussian	λ=32.162 μ=6.3639	λ =29.744 μ =7.9179	λ =50.546 μ =9.0373
Log-Logistic	α =3.3498 β =5.6551	α =3.0998 β =6.8267	α =3.3515 β =8.0679
Logistic	σ=1.5607 μ =6.3639	σ =2.2523 μ =7.9179	σ =2.1068 μ =9.0373
Lognormal	σ =0.52471 μ =1.733	σ =0.57055 μ =1.9212	σ =0.51701 μ =2.0883

Table 1: Contd.,			
Nakagami	m=1.3126 Ω=48.511	m=1.0729 Ω=79.377	m=1.7937 Ω=96.268
Rayleigh	σ=5.0777	σ =6.3176	σ =7.2107
Weibull	α =2.4316 β =7.1671	α =2.208 β =8.862	α=2.4588 β =10.197

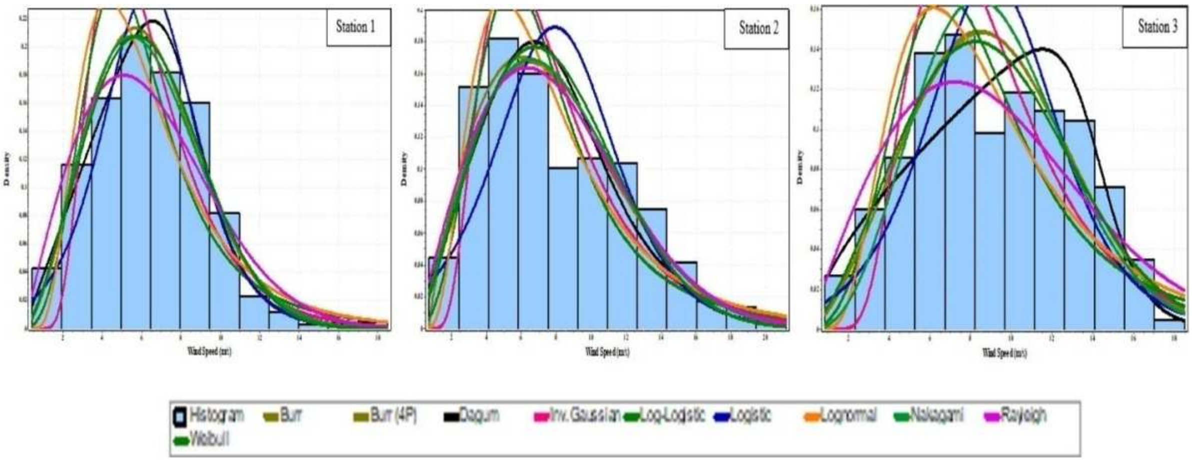

Figure 1: Fitting of Wind Speed Data for Each Station with Various Distribution

Table 2: Results of Goodness of Fit for Various Distributions

Station	Distribution	K-S Test	AD Test	Chi-Squared Test
Station 1	Burr	0.02113	3.0483	62.571
	Burr (4P)	0.02119	3.0125	61.966
	Dagum	0.02248	2.5913	**39.007**
	Inv. Gaussian	0.06729	63.432	355.79
	Log-Logistic	0.08085	39.425	369.02
	Logistic	0.04339	11.843	80.217
	Lognormal	0.08314	41.091	349.82
	Nakagami	0.02605	4.0752	69.142
	Rayleigh	0.05693	27.932	191.91
	Weibull	**0.01859**	**2.0166**	43.306
Station 2	Burr	0.07117	14.36	185.98
	Burr (4P)	**0.05987**	**10.272**	**130.88**
	Dagum	0.06947	19.921	211.19
	Inv. Gaussian	0.07389	49.069	336.28
	Log-Logistic	0.07986	26.096	273.94
	Logistic	0.12408	59.385	433.86
	Lognormal	0.06298	16.771	208.15
	Nakagami	0.07235	15.116	184.61
	Rayleigh	0.0623	11.968	162.95
	Weibull	0.07525	24.807	237.44
Station 3	Burr	0.04315	7.9017	70.617
	Burr (4P)	0.04148	7.3228	**70.535**
	Dagum	0.07349	13.185	114.37
	Inv. Gaussian	0.09348	66.223	279.4
	Log-Logistic	0.09236	34.198	222.24
	Logistic	0.07466	25.873	174.38
	Lognormal	0.08127	34.696	244.02
	Nakagami	0.05902	26.163	150.52
	Rayleigh	0.06297	24.427	172.06
	Weibull	**0.04054**	**5.948**	71.059

The fitting of these distributions are tested using the KS statistics, AD test and Chi-squared test. The results of goodness of fit tests are presented in Table 2. From the results, it is observed that a specific distribution function cannot be suggested for all the stations. The distribution which is having lowest test statistic is recognized as the finest model for the wind speed distribution for each station. From Table 2, it is observed that Weibull distribution gives the best fit for the stations 1 & 3 and for the station 2, Burr (4P) distribution gives the best fit for the wind speed data.

WIND POWER MODELLING

Wind power modeling for the three stations have been done with the 4P logistic and 5P logistic transformations. The constants of the 4P logistic and 5P logistic expressions are solved using Differential Evolution (DE) and Particle Swarm Optimization (PSO) algorithms [10].

Table 3: Constants of 4-Parameter & 5-Parameter Logistic Expression [10]

Station	4P Logistic Expression					5P Logistic Expression				
	Algorithm	a	m	n	τ	a	b	c	d	g
Station 1	DE	973.521	-1.9582	128.8459	1.7667	933.0140	-9.7576	10.7806	-2.990	0.2906
	PSO	1130.04	-0.5586	162.6148	1.8048	1078.08	-7.7892	10.6272	14.124	0.4514
Station 2	DE	406.467	-1.7667	52.2499	2.4473	408.6903	-6.4037	12.7074	-0.918	0.3859
	PSO	471.48	-1.1564	62.8034	2.5795	458.64	-5.4036	13.1328	3.5174	0.4829
Station 3	DE	165.4	-1.7910	118.5	1.98	167.5672	-7.5487	12.1602	-0.144	0.3654
	PSO	198.48	-0.5958	125.2240	2.1888	193.08	-6.4236	12.2208	4.0812	0.523

- Weibull Distribution

From the Table 2, it is observed that the Weibull distribution gives the better fit for the wind speed data of stations 1 & 3. Hence the wind power density and cumulative density functions are derived using the 4P logistic and 5P logistic transformations are as follows:

The probability density function of Weibull distribution is

$$f_x(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right),\ \alpha > 0 \text{, shape parameter and } \beta > 0 \text{, scale parameter}$$

By applying the transformation method, we get the power density function and cumulative density functions as

$$f_p(p) = \frac{\alpha}{\beta}\left(\frac{h^{-1}(p)}{\beta}\right)^{\alpha-1} \exp\left(-\left(\frac{h^{-1}(p)}{\beta}\right)^{\alpha}\right)\frac{d}{dp}h^{-1}(p) \tag{6}$$

$$F_p(p) = 1 - \exp\left(-\left(\frac{h^{-1}(p)}{\beta}\right)^{\alpha}\right) \tag{7}$$

By substituting the corresponding $h^{-1}(p)$ and $\frac{d}{dp}h^{-1}(p)$ of 4P and 5P logistic transformations in the equations (6) & (7) we get the Weibull wind power density and cumulative density functions of 4P and 5P logistic transformations respectively.

- Burr (4P) Distribution

It is observed from the table 2, the Burr (4P) distribution fits well for the wind speed data of station 2. Hence the wind power density and cumulative density functions are derived using the 4P logistic and 5P logistic transformations are as follows:

The probability density function of Burr (4P) distribution is

$$f_x(x) = \frac{\alpha k\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}}{\beta\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{k+1}}$$, $\alpha > 0, k > 0$ are shape parameters & $\beta > 0, \gamma > 0$ are scale parameters.

By applying the transformation method, we get the power density function and cumulative density function as

$$f_p(p) = \frac{\alpha k\left(\frac{h^{-1}(p)-\gamma}{\beta}\right)^{\alpha-1}}{\beta\left(1+\left(\frac{h^{-1}(p)-\gamma}{\beta}\right)^{\alpha}\right)^{k+1}} \frac{d}{dp}h^{-1}(p) \tag{8}$$

$$F_p(p) = 1-\left(1+\left(\frac{h^{-1}(p)-\gamma}{\beta}\right)^{\alpha}\right)^{-k} \tag{9}$$

By substituting the corresponding $h^{-1}(p)$ and $\frac{d}{dp}h^{-1}(p)$ of 4P and 5P logistic transformations in the equations (8) & (9) we get the Burr (4P) wind power density and cumulative density functions of 4P and 5P logistic transformations respectively.

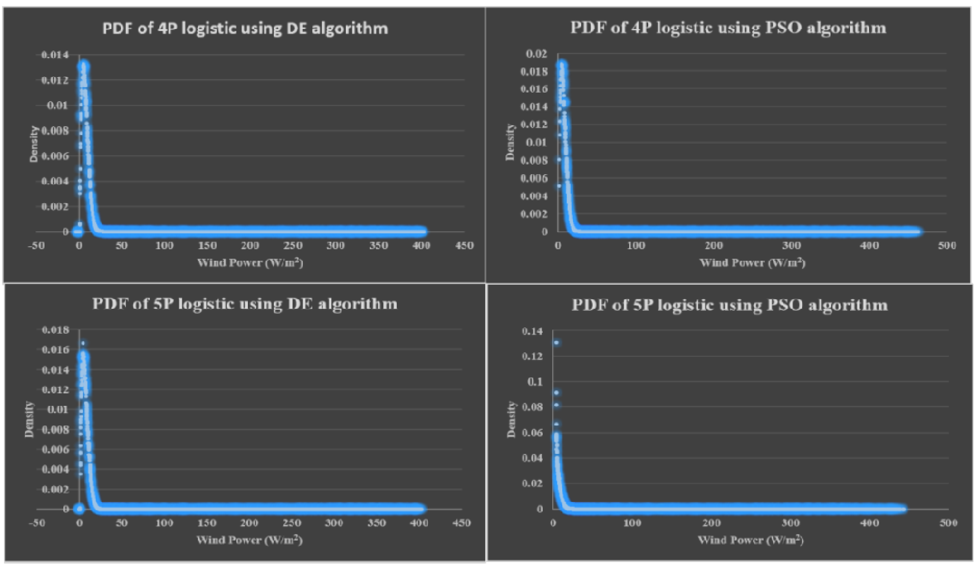

Figure 2: Wind Power Density Function Derived from Weibull Distribution for Station 1

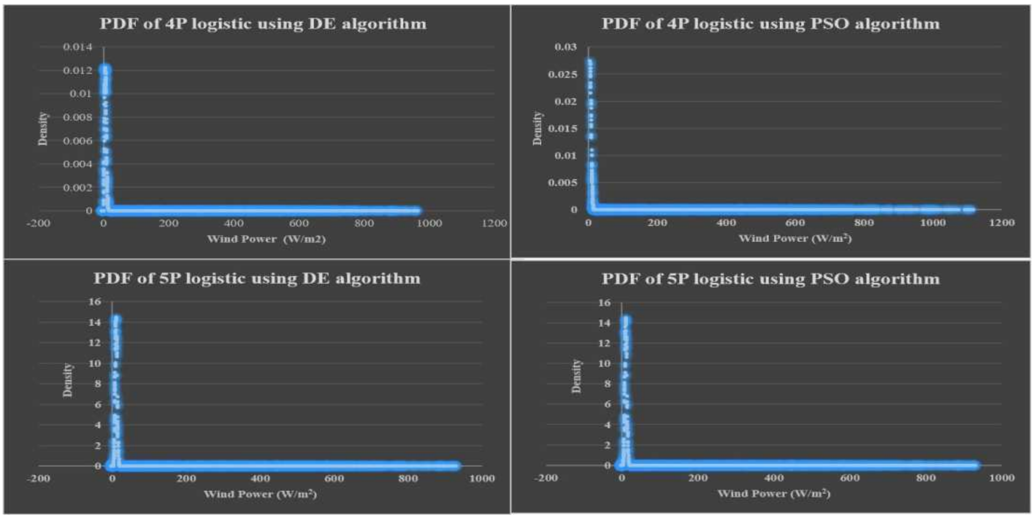

Figure 3: Wind Power Density Function Derived from Burr (4p) Distribution for Station 2

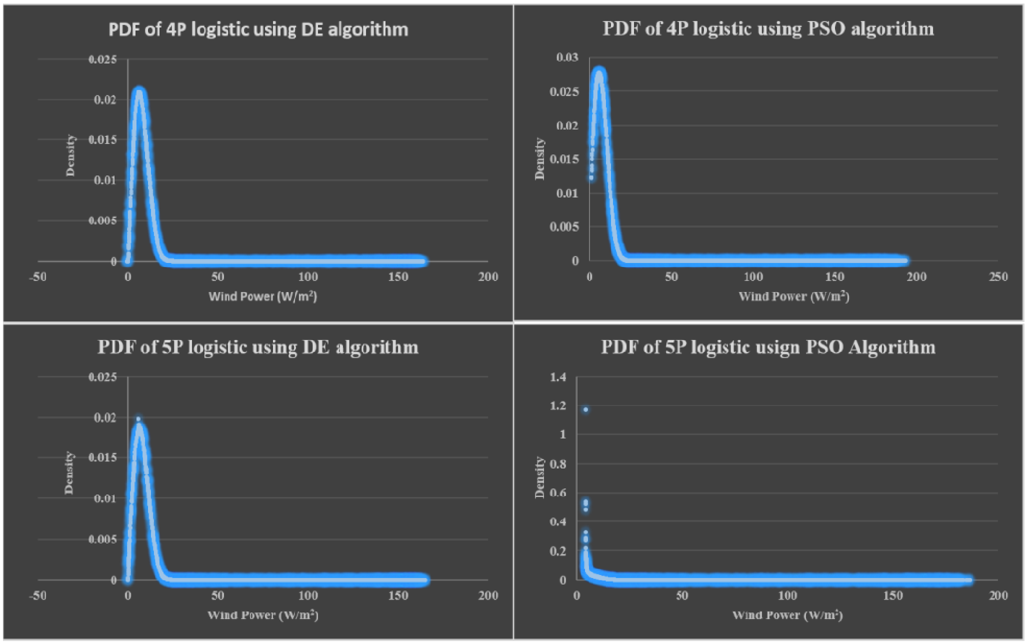

Figure 4: Wind Power Density Function Derived from Weibull Distribution for Station 3

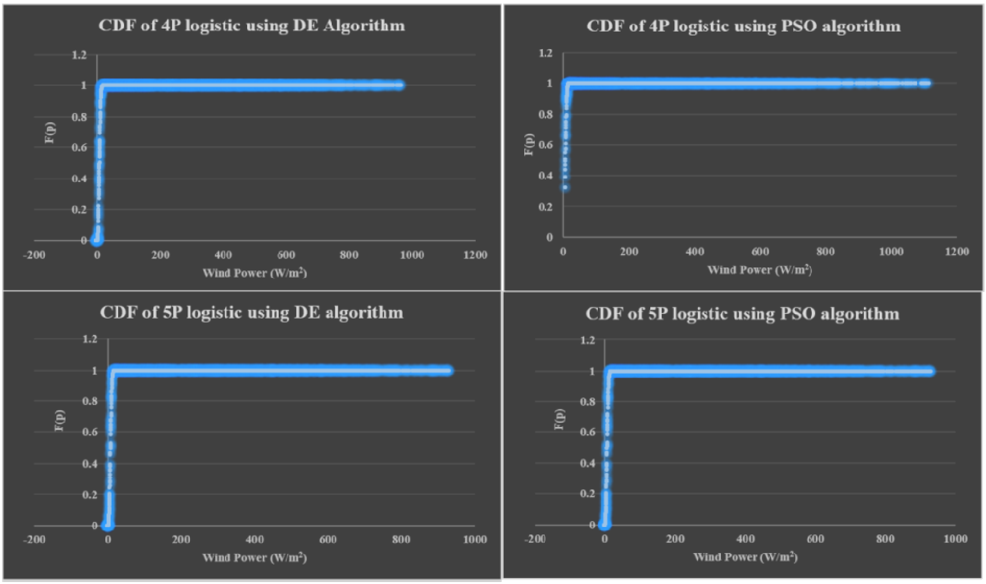

Figure 5: Cumulative Density Function Derived from Weibull Distribution for Station 1

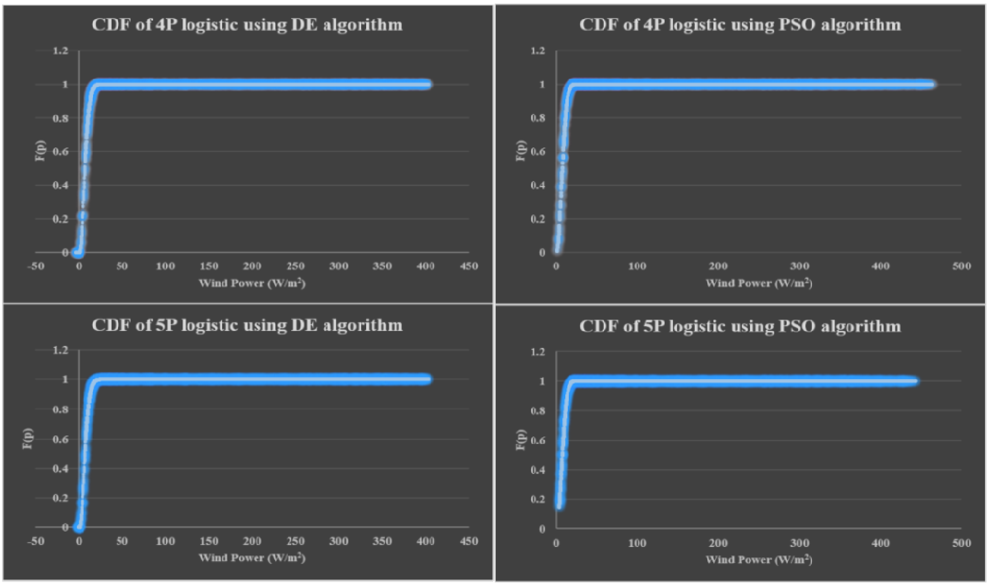

Figure 6: Cumulative Density Function Derived from Burr (4P) Distribution for Station 2

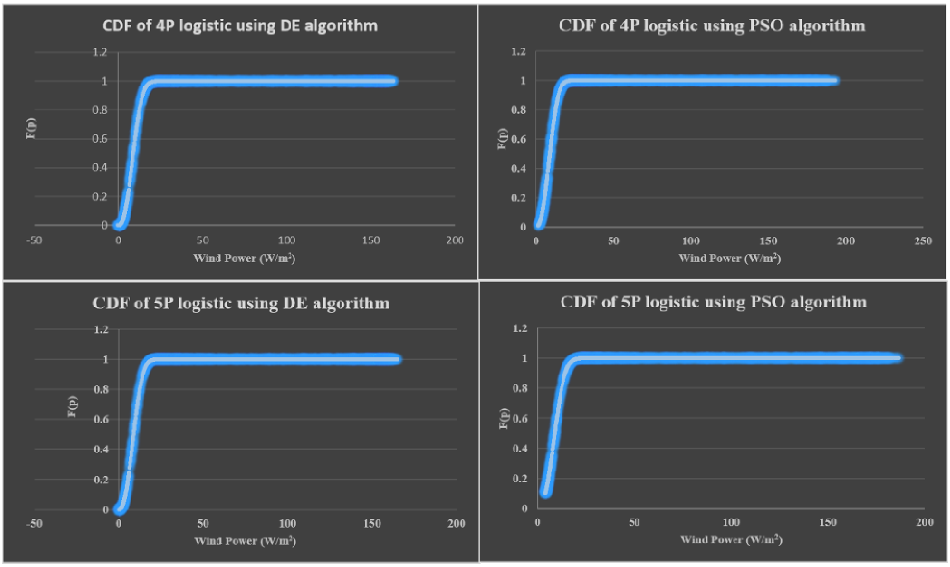

Figure 7: Cumulative Density Function Derived from Weibull Distribution for Station 3

RESULTS AND ANALYSIS

The wind power density curve modelling has been carried out using three sets of wind speed data. The powers of these wind speed data are derived using two transformation techniques with two different algorithms. The wind power derived from these techniques was compared with the observed wind power of these stations. The comparison of empirical and theoretical wind power curves are shown in figure 8. The performance of these transformation techniques has been assessed by the error metrics MAD and RMSE which are defined as follows:

$$MAD = \frac{\sum_{i=1}^{n} |O_i - E_i|}{n} \tag{10}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (O_i - E_i)}{n}} \tag{11}$$

The MAD is the mean absolute deviation between the observed and expected values of power. It shows the amount of deviation that occurs around the mean score. The RMSE is the root mean square error between the observed and expected values of power. It measures how much error there is between the observed and expected data. The MAD and RMSE values of the discussed transformation techniques with two algorithms have been tabulated in Table 4.

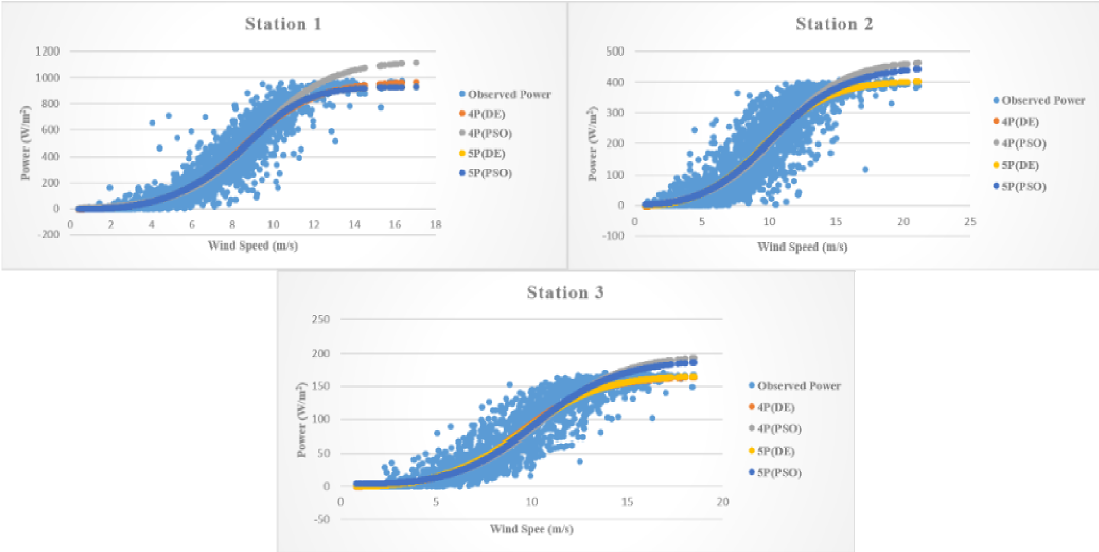

Figure 8: Comparison of Empirical and Theoretical Power Curves

Table 4: Comparison of Transformation Techniques

Station	Transformation Technique	Algorithm	MAD	RMSE
Station 1	4P logistic	DE	66.9535	97.5718
		PSO	69.3364	100.9199
	5P logistic	**DE**	**66.7691**	**97.4914**
		PSO	66.7704	97.4915
Station 2	4P logistic	DE	41.0289	57.2271
		PSO	43.0970	59.2818
	5P logistic	**DE**	**40.9109**	**57.2155**
		PSO	41.7323	57.7812
Station 3	4P logistic	DE	13.4318	19.1279
		PSO	15.3152	20.7257
	5P logistic	**DE**	**13.4238**	**19.0894**
		PSO	14.7064	20.0994

Lower values of MAD and RMSE indicates the better fit. From the Table 4, it is observed that for all the three stations the wind power derived from the 5P logistic transformation technique, whose constants are solved by the DE algorithm gives the better fit for both the error metrics.

CONCLUSIONS

In this article, a comparative case study have been accompanied to fit the best mathematical model for the wind speed data as well as the wind power data of three stations. Evaluation and comparison of ten different distributions for various wind speed data have been done in this study. The wind power density curve has been modelled using four parameter logistic and five parameter logistic transformation methods. From the figure 1 and Table 2, it is observed that the Weibull distribution gives the best fit for the stations 1 & 3, Burr (4P) fits well to the station 2. Hence the transformation methods have been applied to the Weibull and Burr (4P) distributions to derive the wind power from the corresponding wind speed data. The two advanced algorithms namely, Differential Evolution (DE) and Particle Swarm Optimization (PSO) are applied to derive the constants of the logistic expressions. The wind power density curves and cumulative density curves are plotted. The wind power observed from the stations and the powers derived from the two transformation methods are compared. The performance of the transformation methods are analyzed by the error metrics

MAD and RMSE. From the Table 4, it is concluded that the five parameter logistic transformation with the application of Differential Evolution (DE) algorithm provides the better wind power curve modelling for all the examined stations. Precisely modelled wind power curve will certainly progress the performance of wind farms and which will lead to the transformation of wind farm into the wind power plant.

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